A numerical hemodynamic tool for predictive vascular surgery

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Abstract

We suggest a new approach to peripheral vascular bypass surgery planning based on solving the one-dimensional governing equations of blood flow in patient-specific models. The aim of the present paper is twofold. First, we present the coupled 1D-0D model based on a Discontinuous Galerkin method in a comprehensive manner, such as it becomes accessible to a wider community than the one of mathematicians and engineers. Then we show how this model can be applied to predict hemodynamic parameters and help therefore clinicians to choose for the best surgical option bettering the hemodynamics of a bypass. After presenting some benchmark problems, we apply our model to a real-life clinical application, i.e a femoro-popliteal bypass surgery. Our model shows very good agreement with in vitro measurements and post surgical reports.

Key words: Blood flow, vascular surgery, multiscale modeling, hyperbolic system, Discontinuous Galerkin

1 Introduction

Atherosclerosis, the most common type of cardiovascular disease, gives rise to the thickening of large- and medium- size arteries and leads to tissue malperfusion. When occurring in the lower limb network, calf pain when walking

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(claudication) and then at rest are the first symptoms. If not treated, the patient may suffer from leg ulcers. A common method to treat an arterial narrowing (stenosis) is to dilate or stent it. When a long arterial segment is occluded, a bypass remains the only surgical procedure to overcome the lesion and relieve the patient’s symptoms. The vascular grafts most often used to perform the bypasses are the prosthetic ones made of polytetrafluoroethylene (PTFE) or polyester (dacron) and the patient’s superficial leg vein (internal saphenous vein).

To evaluate the severity of the disease, to locate the occluded arterial segments and to get the morphology of the runoff vessels, duplex ultrasound and peripheral angiography (by direct arterial puncture, computed tomography or magnetic resonance imaging) are performed. Vascular surgery planning is commonly based on these morphological data and on clinical studies from the literature [1–3] rather than on predictive computations of different surgical options. The choice of the bypass should give the best hemodynamic characteristics in order to better the hemodynamics of bypasses.

Mathematical models for cardiovascular systems are largely used to simulate blood flow in arteries and to predict hemodynamical patterns in physiological and pathological conditions. Some applications of the model have already been used for predictive vascular surgery [4–8] but this is not yet of a common practice in the medical community.

As far as those models are concerned, according to the specific scale of phenomena to be studied, various degrees of simplifications can be considered going from three dimensional fluid structure interaction problems to zero dimensional models. Three-dimensional models which aim to study local flow phenomena [9, 10] are based on the numerical approximation of the incompressible Navier-Stokes equations, possibly accounting for compliant vessels. However, from a computational point of view, three dimensional numerical simulation of the whole circulatory system is at the moment unaffordable. Reduced one-dimensional models of the human arterial networks [11–14] are based on the resolution of a system of non-linear partial differential equations that were first derived by Euler in 1775 [15]. Those reduced models are an efficient way to analyze quickly quantitative features of human artery flows and their justification arises due to the long wave length associated with the system in comparison to the length of the arteries. With further simplifications, one can obtain zero-dimensional models (lumped parameter models) [16–18]. These lumped models of the circulatory network are based on electrical analog of a hydraulic description of the system. They offer the advantage of simply representing global characteristics such as the heart, the network of capillaries and the venous bed.

In order to compute global effects, several coupled models have been sug-
gested in the literature, in particular the coupling between 3D and 1D models reported in [19], the coupling between 3D and 0D [20] and the coupling between 1D and 0D [21,22].

Within the context of vascular surgery planning, we aim to develop an efficient tool for predicting general hemodynamic features within a few minutes of computational time. Therefore, we have chosen to develop a coupled 1D-0D model. In this paper, we suggest a new self-consistent approach to vascular surgery planning based on an one-dimensional discontinuous Galerkin method coupled with a lumped parameter model (section 2,3). We aim to provide the reader with a thorough collection of benchmark problems to provide a better understanding of the different aspects of the mathematical model, the numerical method and the hemodynamics of arteries. We also present a relevant clinical application of a femoro-popliteal bypass surgery (section 4) that shows how this model will help the clinicians to choose for the best surgical option bettering the hemodynamic conditions.

2 Mathematical model

The derivation of the one-dimensional governing equations for the the blood flow in variables $A, u, p$ can be found in several articles [12, 13, 23] . Hence, in this paper, we state them without any new derivation. Nevertheless, we shortly repeat the main assumptions made. Firstly we assume that blood in reasonable large vessels can be modeled as incompressible, Newtonian fluid with constant density $\rho$ and constant dynamic viscosity $\nu$ [24]. The Reynolds number based on the vessel diameter $Re = ud/\nu$ is below 2000 in all vessel segments of the cardiovascular system, so that the flow can be assumed to be laminar [24]. The wave velocity may take values as low as $5 \text{ m/s}$ in the aorta, rising to values around $20 \text{ m/s}$ in less distensible peripheral arteries or to $35 \text{ m/s}$ in strongly deformed tubes. However, peak flow velocities are much smaller, generally around $1 \text{ m/s}$, while they can reach $6 \text{ m/s}$ in segments of severe deformation.

2.1 Governing equations

The one-dimensional equations of an incompressible and Newtonian fluid within an elastic tube result from the conservation of mass and momentum and take the following form [13]:

$$\frac{\partial A}{\partial t} + \frac{\partial (Au)}{\partial x} = 0,$$  \hspace{1cm} (1)
\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{1}{\rho} \frac{\partial p}{\partial x} + K_R u = 0,
\] (2)

where \(x\) is the axial coordinate along the vessel, \(t\) is the time, \(A = A(x,t)\) is the cross sectional area, \(u(x,t)\) is the average axial velocity, \(p(x,t)\) is the average internal pressure over the cross section, \(\rho\) is the blood density, and \(K_R\) is a positive quantity which represents the viscous resistance of the flow per unit length of tube. Assuming a constant and flat velocity profile, we take \(K_R\) to be equal to \(22\pi \nu\) [25, 26]. This flat velocity profile corresponds to an observed profile from a Doppler ultrasonography (US) examination (see the uniform coloring of the arteries in Fig. 1).

Fig. 1. Doppler US performed after a femoro-popliteal bypass surgery at two different locations: popliteal artery and Dacron graft. Both Doppler color imaging show a flat velocity profile.

This system of equations is closed with a pressure-area relation, \(p = \mathcal{F}(A(x,t), x, t)\). We adopt the relation previously used in [12–14], which assumes a thin, homogeneous and elastic arterial wall:

\[
p(x,t) = p_0 + \beta \left( \sqrt{A} - \sqrt{A_0} \right), \quad \beta = \beta(x) = \frac{4\sqrt{\pi h_0 E(x)}}{3A_0}.
\] (3)

where \(A_0\) and \(h_0\) are the sectional area and wall thickness at the reference state \((p_0, U_0)\). We assume \(p_0 = 0\), i.e. an atmospheric external pressure and \(U_0 = 0\), an initial state at rest. \(E\) is the Young’s modulus.

For our clinical applications, the parameter \(\beta\) is however computed in a straightforward manner, from the velocity wave (see Eq. (6)) rather than from Eq. (3). The pulse wave velocity is measured from a Pulse Trace PWV (Micro Medical). PWV uses two Doppler probes distant from a known length to detect the onset of flow in the arteries. Doppler pulses are recorded sequentially and compared using the R-wave of the ECG.
2.2 The characteristic system

Due to the fact that physiological conditions of the arterial system are only weakly non-linear, many characteristics can be captured by linearizing the system. The characteristics are applicable to compute upwind variables or to derive suitable boundary conditions. A quasi-linear first-order characteristic system for the variables $A$ and $u$ can be written so that:

$$\frac{\partial U}{\partial t} + H(U) \frac{\partial U}{\partial x} = S(U) \quad (4)$$

where:

$$\begin{bmatrix} A \\ u \end{bmatrix}_t + \begin{bmatrix} u & A \\ c^2/A & u \end{bmatrix} \begin{bmatrix} A \\ u \end{bmatrix}_x = \begin{bmatrix} 0 \\ f \end{bmatrix} \quad (5)$$

$c$ is the speed of pulse wave propagation and $f$ is the forcing term:

$$c = \sqrt{\frac{A}{\rho} \frac{\partial p}{\partial A}} = \sqrt{\frac{\beta}{2\rho}} A^{1/4}, \quad f = \left[ -K_R u - \frac{1}{\rho} \left( \frac{\partial p}{\partial \beta} \frac{\partial \beta}{\partial x} + \frac{\partial p}{\partial A_0} \frac{\partial A_0}{\partial x} \right) \right]. \quad (6)$$

Under the assumption that $A > 0$, which is indeed a necessary condition to have a physically relevant solution, the matrix $H(U)$ has two real eigenvalues $\lambda_{1,2}$ and a complete set of eigenvectors $L(H)$:

$$\lambda(H) = \begin{bmatrix} u + c \\ u - c \end{bmatrix}, \quad L(H) = \begin{bmatrix} A/c & -A/c \\ 1 & 1 \end{bmatrix}. \quad (7)$$

We have $H = L \Lambda L^{-1}$, with $\Lambda(H)$ the diagonal eigenvalue matrix.

As previously mentioned, the wave speed of the non linear system $c$ is much larger than the velocity of blood in arteries ($u \ll c$). Hence, the system is subsonic and we have $\lambda_1 > 0$ and $\lambda_2 < 0$.

At this point, we make the assumption of the absence of viscous forces ($K_R = 0$) for resolving our characteristic system. After introducing a change of variables ($\partial U W = L^{-1}$) we can transform equation (4) into a system of decoupled scalar equations [13]:

$$\frac{\partial W}{\partial t} + \Lambda \frac{\partial W}{\partial x} = 0, \quad (8)$$

The characteristic variables $W$ can be determined by integrating the differential system. For a particular choice of integration domain, we may write the characteristic variables as follows:

$$W_{1,2} = u \pm 4c = u \pm 4 \sqrt{\frac{\beta}{2\rho}} A^{1/4}. \quad (9)$$
With those definitions of the characteristic variables, we may write the variables \((A, u)\) in terms of the \(W_1\) and \(W_2\). Those variables are denoted \(A^{up}\) and \(u^{up}\), to note for upwind variables:

\[
A^{up} = \left[\frac{(W_1 - W_2)}{4}\right]^4 \left(\frac{\rho}{2\beta}\right)^2, \quad u^{up} = \frac{W_1 + W_2}{2}.
\] (10)

### 3 Discrete blood flow model

#### 3.1 The discontinuous Galerkin method

We have chosen to discretize our system of equations by a Runge-Kutta discontinuous Galerkin method (RK-DG). The RK-DG scheme is very suitable for the hyperbolic blood flow equations [27–29] because it can propagate waves of different frequencies without suffering from excessive dispersion and diffusion errors [30,31].

In this section, we briefly describe the RK-DG method. More details of the algorithm can be found in [32] where the RK-DG method is used to discretize the hyperbolic level set equation. To start, we write the system (1)-(2) in conservative form as:

\[
\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial x} = S(U)
\] (11)

with

\[
U = \begin{bmatrix} A \\ u \end{bmatrix}, \quad F = \begin{bmatrix} Au \\ u^2 + \frac{p}{\rho} \end{bmatrix}, \quad \text{and} \quad S = \begin{bmatrix} 0 \\ -K_R u \end{bmatrix}.
\] (12)

Next, we discretize the domain of each artery, \(\Omega = [0, l]\) into a mesh of \(N_{el}\) elements \(\Omega_e = [x_e, x_{e+1}]\). In each element, we discretize the space of unknown using polynomials of order equal to \(p\).

For that, we consider a reference segment \(\xi \in (-1, 1)\); the unknown \(U^e\) in segment \(e\) going from \(x_e = (e - 1)/N\) to \(x_{e+1} = e/N\) is approximated as

\[
U^e(x(\xi), t) = \sum_{i=0}^p U^e_i(t) N_i^p(\xi) \quad \text{with} \quad x(\xi) = x_e \frac{1 - \xi}{2} + x_{e+1} \frac{1 + \xi}{2}.
\] (13)

Here, \(N_i^p\) denotes the \(i^{th}\) Lagrange polynomial of order \(p\). Figure 2 shows the one dimensional discretization.

The weak form of the system is then obtained by multiplying (11) by a test function \(N^p\), integrating over the domain \(\Omega\) and using the divergence theorem:
Since the DG method allows discontinuities at the interface, the flux $F^e$ is not uniquely determined on it and a flux formula has to be supplied to complete the discretisation process. An appropriate upwind flux ($F_{up}$) is chosen and discussed in the next section.

### 3.2 Flux Upwinding

The upwind fluxes $F_{up}$ required at the interfaces of each elemental region $\Omega_e$ are computed through the solution of a Riemann problem that takes into account the characteristic information arriving at both sides of the interface and neglects the characteristic information moving away. At a time $t$, each interface separates two constant states $(A_L, u_L)$ and $(A_R, u_R)$ (Figure 3), with their corresponding values of $\beta$. The appropriate characteristic information at the interface is given by:

\[
W_1 = u_L + 4\sqrt{\frac{\beta_L}{2\rho}} A_L^{1/4} \\
W_2 = u_R - 4\sqrt{\frac{\beta_R}{2\rho}} A_R^{1/4}
\]

If $\beta$ is continuous ($\beta_L = \beta_R$), $F_{up}$ is unique at the interface. It is computed by posing $F_{up} = F(A_{up}, u_{up})$, where $(A_{up}, u_{up})$ are computed from $W_1$ and $W_2$ by equations (10) and applying equation (12).

If now, there are discontinuities in $\beta$ across the interface, equations (10) do not apply and $F_{up}$ is not unique anymore at the interface. Equations (15) and (16) need to be supplied with additional information. A reasonable choice taken by [13] is to assume conservation of mass and of total pressure $P_t$ at the interface.
Fig. 3. Layout of the Riemann problem that calculates the upwind states \((A_{L}^{up}, u_{L}^{up})\) and \((A_{R}^{up}, u_{R}^{up})\) originated from the discontinuity between two initial states \((A_{L}, u_{L})\) and \((A_{R}, u_{R})\) (taken from [25]).

\[
Q = A_{L}u_{L} = A_{R}u_{R} \tag{17}
\]

\[
P_{l} = \rho \frac{u_{L}^{2}}{2} + p_{L} = \rho \frac{u_{R}^{2}}{2} + p_{R} \tag{18}
\]

Finally, equations (15), (16), (17) and (18) are solved by means of an iterative Newton-Raphson method to obtain \((A_{L}^{up}, u_{L}^{up})\) and \((A_{R}^{up}, u_{R}^{up})\). Subsequently, the upwind fluxes at each side of the interface are calculated as:

\[
F_{L}^{up} = F(A_{L}^{up}, u_{L}^{up}) \quad \text{and} \quad F_{R}^{up} = F(A_{R}^{up}, u_{R}^{up}).
\]

### 3.3 Bifurcation treatment

At a bifurcation, we have six unknowns: \((A_{L}^{up}, u_{L}^{up})\) in the parent vessel, \((A_{R1}^{up}, u_{R1}^{up})\) in the upper daughter vessel and \((A_{R2}^{up}, u_{R2}^{up})\) in the lower daughter vessel (Figure 4).

Fig. 4. Layout of the Riemann problem in the case of an arterial tree bifurcation.

The same procedure that has been described can be applied to find those upwind variables. They are obtained by solving a non-linear system of six al-
gebraic equations using a Newton-Raphson method. The first three equations are obtained by imposing that the incoming characteristic variables in each vessel should remain constant, i.e. $W_1, W_{21}$ and $W_{22}$ (see Figure 4).

The last three equations are obtained from the continuity of mass and total pressure at the interface:

\[
Q = A_L u_L = A_{R1} u_{R1} + A_{R2} u_{R2} \tag{19}
\]
\[
P_t = \rho \frac{u_L^2}{2} + p_L = \rho \frac{u_{R1}^2}{2} + p_{R1} \tag{20}
\]
\[
P_t = \rho \frac{u_L^2}{2} + p_L = \rho \frac{u_{R2}^2}{2} + p_{R2} \tag{21}
\]

We note that with this approach, the bifurcation is defined only at one point and that the energy is conserved.

3.4 In- and Outflow Boundary conditions

Two types of boundary conditions can be prescribed: either reflecting boundaries or absorbing boundaries depending on the physics we want to model. Those type of boundary conditions should reflect the physics of the clinical application (opening and closure valves, etc...)

For reflecting boundary conditions, we enforce at the interface either a time dependent area $A_{bc}(t)$ by imposing $A^{up} = A_{bc}(t)$ (with $u^{up} = u_R$), or a time dependent velocity $u_{bc}(t)$ by imposing $u^{up} = u_{bc}(t)$ (with $A^{up} = A_R$) or even a time dependent flow rate $Q_{bc}(t)$ by imposing $u^{up} = Q_{bc}(t)/A_R$ (with $A^{up} = A_R$).

For absorbing boundary conditions, the desired incoming state is weakly enforced through the upwind characteristic information whilst the outgoing characteristic remains constant.

- To weakly enforce the velocity $u_{bc}(t) = u_L$ at the inlet of an arterial domain, we rewrite $W_1$ in terms of $W_2$ and $u$: $W_1 = 2u - W_2$ and assume that $W_2$ remains constant, i.e.:
  \[
  W_1 = 2u_{bc} - W_20 \tag{22}
  \]
- To weakly enforce the pressure $p_{bc}(t) = p_L$ at the inlet of an arterial domain, we rewrite $W_1$ in terms of $W_2$ and $p$: $W_1 = W_2 + 4\sqrt{\frac{2}{\rho}} \left( \sqrt{(p - p_0)} + \beta \sqrt{A_0} \right)$ and assume that $W_2$ remains constant, i.e.:
  \[
  W_1 = W_20 + 4\sqrt{\frac{2}{\rho}} \left( \sqrt{(p_{bc} - p_0)} + \beta_R \sqrt{A_{R0}} \right) \tag{23}
  \]
We note that this same procedure can be applied to enforce strongly or weakly a value at the outlet of an arterial domain. However, the imposed values should account for the rest of the arterial bed, the capillaries and the venous bed. There are different ways to account for this, starting from a pure resistive load where the outflow is proportional to the pressure, over three and four-element windkessel models [33] to a structured tree outflow condition suggested in [34], or to a closed-loop lumped parameter model [35].

We have chosen to use a reasonable three-element windkessel model given in [34]. The main advantage of this model is to consider the compliant-capacitive effects due to microvessels and arterioles. The lumped analog electrical circuit is shown in Fig. 5(b).

For the three element windkessel model (RCR), the analog circuit is governed by:

\[ p_{in} + R_{\mu 2} C \frac{dp_{in}}{dt} = p_{out} + (R_{\mu 1} + R_{\mu 2}) Q_{in} + R_{\mu 1} R_{\mu 2} \frac{dQ_{in}}{dt}. \]  

This system can be considered as the sum of a resistance model \( R_{\mu 1} \) coupled to a 1D terminal branch, followed by a \( CR_{\mu 2} \) model. According to the study in [8], we consider \( R_{\mu 1} = \rho c_0 L / A_0 L \) to allow any incoming wave from the 1D terminal branch to reach \( R_{\mu 2} \) and \( C \), without being reflected by \( R_{\mu 1} \).

The RCR model is coupled to the 1-D terminal branch by assuming that the state \((A_{up}, u_{up})\) satisfies the equations of the simple electrical circuit (see Fig. 5(a)) with \( R_{\mu} = R_{\mu 1} \) and \( p_{out} = p_C \), where \( p_C \) is the pressure across the capacitance. More precisely, the value of \( A_{up} \) is evaluated by solving a non-linear equation in \( A_{up} \) that arises from the combination of the equation of the pure resistive model

\[ Q_{up} = A_{up} u_{up} = \frac{p(A_{up}) - p_{out}}{R_{\mu}}, \]  

and of the equation of the invariance of \( W_1 \) from the end point of the 1D terminal artery to the outlet: \( W_1(A_L, u_L) = W_1(A_{up}^p, u_{up}^p) \).

At every time step \( n \), \( p_C \) is determined by solving a first-order time discreti-
sation of the conservation of mass in the capacitance:

\[ p_{C_{n}}^{n} = p_{C_{n-1}}^{n-1} + \frac{\Delta t}{C} \left( Q_{in_{n-1}}^{n-1} - Q_{out_{n-1}}^{n-1} \right), \]  

(26)

with \( Q_{in_{n-1}}^{n-1} = (A_{up})^{n-1} (u_{up})^{n-1} \) and \( Q_{out_{n-1}}^{n-1} = \frac{p_{C_{n-1}}^{n-1} - p_{out}}{R_{p2}} \).

4 Applications

In the following section, we first analyze the effect of the arterial bifurcation. We then apply our method to study the effect of a bypass in an arterial tree based on the physiological references given by Stergiopulos and al. [11]. The last example is a clinical application that validates the numerical results against the clinical application of a femoro-popliteal bypass.

In all the simulations, each physical segment of length \( L \) is divided into \( N_{e} \) elements and within each element, we use polynomials of order \( p = 6 \) to approximate our variables \((A, u)\). All the simulations take less than 3 minutes to run on a MacBook pro Intel Core 2 Duo.

4.1 Arterial tree bifurcation

The first benchmark problem is taken from [36], where they consider three segments (a parent vessel and two daughters vessels) connected to make a single bifurcation. The parameters used for the simulation are shown in Table 4.1.

<table>
<thead>
<tr>
<th>Artery</th>
<th>L (cm)</th>
<th>( \beta ) (g/(s^2 cm^2))</th>
<th>( A_{0} ) (cm^2)</th>
<th>( R ) (g/(cm^4 s^1))</th>
<th>( C ) (cm^4/(s^2 g))</th>
<th>( N_{e} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parent</td>
<td>20</td>
<td>32.497</td>
<td>0.785</td>
<td>-</td>
<td>-</td>
<td>10</td>
</tr>
<tr>
<td>Daughter</td>
<td>20</td>
<td>79.602</td>
<td>0.1309</td>
<td>0</td>
<td>0</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 1
Parameters used for the Bifurcation test case.

With those parameters and using equation (6), we observe that the wave speed is \( c = 120 \text{ cm/s} \) both in the parent and daughter vessels, which is a plausible physiological value for a large artery.

A time step of \( \Delta t = 10^{-5} \) is chosen and the viscosity is set to zero for this test case. Following [36], we consider the flow to be initially at rest and impose an absorbing velocity boundary condition:

\[ u(0, t) = 0.5 \exp(-5000(t - 0.05)^2). \]  

(27)
The outflow artery is modeled by a null windkessel model \((R = 0, C = 0)\), so that any incoming wave is completely absorbed by the outflow boundary.

Figures 6 and 7 show the solution of the pressure wave at different times, the pressure time history at the beginning \((A)\), middle \((B)\) and end \((C)\) of the parent vessel as well as at the middle \((D)\) and end \((E)\) of the daughter vessel. On figure 7, we observe that the incoming wave has a magnitude of 60, the reflected wave of 30 and the transmitted wave of 90. This corresponds to a reflection coefficient at the bifurcation of 0.5, and this is in agreement with a linear reflecting coefficient analysis (see [13] for more details). Besides looking at the waves and the position in times of the waves at Fig. 7, we observe that the wave propagates at the right speed of 120 \(cm/s\). It can be seen from these figures that our method exhibits good conservation properties since the pressure wave is transported without suffering from any numerical dissipation.

![Fig. 6. Solution of the pressure wave at different times](image)

![Fig. 7. Pressure time history at different locations in the arterial tree bifurcation.](image)

We also performed a comparison of our code with the following two benchmark problems:

- Stented artery [13,37];
- Pulse wave propagation in the aorta [25].
It should be noted that our code also provides the same results as those obtained by the model of Sherwin et al. [13] and Formaggia et al. [37].

4.2 Human arterial tree with bypass

In order to show the capabilities of our numerical method, we aim to model the arterial flow in an arterial tree made of 55 arteries (see Figure 9) and 27 arterial bifurcations.

The physiological data used in the model are based upon the original data collected by Westerhof et al. [16] and Stergiopulos et al. [11]. The cross-sectional diameter decreases approximately linearly and the elastic modulus increases linearly in a human arterial tree (see Figure 8).

The 46th artery (left femoral artery) has been replaced by a bypass graft made of Dacron of diameter 8 mm. The value of the elasticity parameter $\beta$ is set to $1.20 \times 10^7 g/(cm^2s^2)$, computed from the measurement of the pulse wave velocity. This value is about 4 times higher than the original elasticity parameter in the left femoral artery provided by [11]. We confirmed this original data by measuring the mean value of this parameter among a sample from the healthy population.

At the inlet of the aorta, we impose a reflecting flow rate boundary condition

$$Q_{bc}(t) = 311.5 \sin\left(\frac{\pi t^*}{0.25}\right) H(t - t^*),$$

where $t^* = t - 0.8 \ n$, $n = \text{floor}(t/0.8)$ and $H$ is the Heaviside function. This flow rate corresponds to a mean cardiac output of 3.8 $l/min$ measured in [38]. At the outlets of the arteries, we use RCR lumped circuits with the parameters being those in [11].

Figures 9 represent the pressure wave propagating through the whole arterial tree. The initial impulsion of the heart is represented with the inlet boundary condition in the aorta. From the left figures to the right ones, we can observe the wave propagation in the systemic arteries. The introduction of a synthetic graft in the left leg increases the wave propagation in the lower left limbs, as it can be compared to the right leg. Indeed, as that material presents a greater stiffness than the human arterial wall, the wave travels faster.
4.3 Femoro-popliteal bypass

We now assess the pertinence of the numerical model against a clinical test case. We aim to show that the model can predict the velocity and pressure profiles for this clinical application. It is indeed of clinical interest to determine the presence of sufficient flow through the bypass graft to decide whether or not the bypass graft is efficient and will not have a tendency for occlusion. According to [39], a low flow velocity for systolic peak pressure (< 45 cm/s) threatens graft patency.

Figure 10 shows the arteriography for a patient specific femoro-popliteal graft with the computational domain. The computational domain goes from the common femoral artery to the tibial arteries.

At the inlet of the common femoral artery, the velocity profile measured with Doppler US is prescribed. At the outlet of the deep femoral and popliteal artery, a RCR lumped parameter model is coupled with the total peripheral resistances \( R = R_{\mu 1} + R_{\mu 2} \) and compliances \( (C) \) shown in Table 2. The initial
conditions are \((A_0, u_0)\). The diameter of the arteries has been measured during diastole \(p_{\text{diast}}\) (60 mmHg) with Doppler ultrasonography (US) and is denoted \(A_{\text{diast}}\). From this diameter, we estimate the reference diameter \(A_0\) using the tube law (3):

\[
A_0 = \left( \sqrt{A_{\text{diast}}} - \frac{(p_{\text{diast}} - p_0)}{\beta} \right)^2.
\]

(29)

At first, the simulation is run with a Dacron bypass of 8 mm, which is the option that was taken by the surgeons based on their experience and on empirical data.

The value of \(R\) is computed as the ratio between the mean pressure and the mean flow rate: \(R = (\bar{p} - p_{\text{out}})/\bar{Q}\). The mean flow rate is computed from Doppler ultrasonography (US) (a non-invasive exam performed prior to surgery) and the mean pressure is computed from data provided by a pressure catheter (an invasive exam performed during surgery). The venous pressure \(p_{\text{out}}\) is set to zero. The mean pressure is obtained by integrating over one cardiac cycle \(T\) the time history of the pressure \(\bar{p} = \frac{1}{T} \int_0^T p\, dt\). This value is quite close to the mean arterial pressure (MAP) that is usually computed by the doctors: \(\text{MAP} = p_d + (p_s - p_d)/3\), where \(p_s\) is the systolic pressure and \(p_d\) the diastolic pressure. The compliance \(C\) is determined by fitting an exponential function during the last two third of diastole in the in-vivo pressure (the time constant of the exponential decay is \(RC\) [40, 41]).
Fig. 10. Arteriography of the femoro-popliteal flow before and after the by-pass surgery (a) and computational mesh with lumped windkessel models as outflow boundary conditions (b).

Figure 11 shows the pressure and velocity time histories when the solution has become periodic (after 5 cardiac cycles) and compares this solution with measurements performed by Doppler US after the surgery. We observe that the peak systolic velocity in the by-pass is 38 cm/s, which is quite a low value for an optimum graft [39]. It was observed that the realized graft got completely obstructed three months later.

In order to provide the clinicians with a tool to choose for the best surgery options, we aim to introduce some first results from our analysis. Further discussions on sensitivity analysis and parameter identification will be carried out soon in coming articles.
### Table 2
Parameters for the numerical simulation of a patient specific femoro-popliteal graft.

<table>
<thead>
<tr>
<th>Artery</th>
<th>L (cm)</th>
<th>$\beta$ ($10^3 g/(s^2 cm^2)$)</th>
<th>$A_0$ ($cm^2$)</th>
<th>$R$ ($g/(cm^4 s^1)$)</th>
<th>$C$ ($cm^4/(s^2 g)$)</th>
<th>$N_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Common Femoral</td>
<td>5</td>
<td>1906</td>
<td>0.309</td>
<td>-</td>
<td>-</td>
<td>3</td>
</tr>
<tr>
<td>Deep Femoral</td>
<td>12.6</td>
<td>2049</td>
<td>0.248</td>
<td>16325</td>
<td>4.31e-5</td>
<td>6</td>
</tr>
<tr>
<td>Dacron Graft</td>
<td>33</td>
<td>12000</td>
<td>0.502</td>
<td>-</td>
<td>-</td>
<td>12</td>
</tr>
<tr>
<td>Popliteal</td>
<td>9</td>
<td>2623</td>
<td>0.105</td>
<td>-</td>
<td>-</td>
<td>4</td>
</tr>
<tr>
<td>Anterior Tibial</td>
<td>32</td>
<td>3223</td>
<td>0.082</td>
<td>39235</td>
<td>4.31e-5</td>
<td>8</td>
</tr>
<tr>
<td>Posterior Tibial</td>
<td>34</td>
<td>4923</td>
<td>0.092</td>
<td>43235</td>
<td>4.31e-5</td>
<td>8</td>
</tr>
</tbody>
</table>

Fig. 11. Femoro-popliteal graft. Comparison of pressure (a) and velocity (b) time histories between (n) numerical simulation and (p) patient specific measured data.

We try here to compute different options of the femoro-popliteal bypasses such as the area, material and shape. Figure 12 shows the velocity time histories for four different grafts: constant-diameter Dacron of 8 and 6 mm, tapering Dacron $8 - 4$ mm and theoretical tapering homograft $6 - 4$ mm. We see for instance that using the tapering Dacron graft would approximately double the peak systolic velocity and enhance the end systolic velocity, which are better criteria according to previous clinical results [39,42].

### 5 Conclusion

We have presented a self-consistent approach for vascular surgery planning. The method consists of a one-dimensional model coupled with a lumped parameter model for the simulation of blood flow through arteries. Some bench-


Fig. 12. Velocity time history in the femoro-popliteal bypass for different grafts. Problems have been presented. They show the good conservation properties of our numerical method and provide a better understanding of the hemodynamics of healthy and pathological conditions. The application to femoro-popliteal bypasses shows a good agreement with flow velocities observed with in vitro measurements. Besides, we have briefly given an overview of the capabilities of this simple and very fast model for predictive vascular surgery planning.

References


